Solution proposal seminar 1

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Problem 1

Let $u(C_1)$ utility if type 1 (early consumer), $u(C_2)$ utility if type 2 (late consumer). Specifically $u(C) = \frac{1}{1-s}C^{1-s}$ Let λ the fraction of type 1 consumers, whereas $1-\lambda$ the fraction of type 2 consumers. Then:

$$\max_{C_1,C_2} \lambda u(C_1) + (1-\lambda)u(C_2) \tag{1}$$

$$\text{s.t.}$$

$$\lambda C_1 = 1 - I \tag{2}$$

$$(1-\lambda)C_2 = RI \tag{3}$$

Then recall the efficiency locus by combining (2) and(3), namely:

$$\lambda C_1 + \frac{1-\lambda}{R}C_2 = 1 \tag{4}$$

Hence the Lagrangian is:

$$\mathcal{L} = \lambda u(C_1) + (1-\lambda)u(C_2) - \mu(\lambda C_1 + \frac{1-\lambda}{R}C_2 - 1) \tag{5}$$

$$\Rightarrow \frac{u'(C_1)}{u'(C_2)} = R \tag{6}$$

Now let's find the derivatives when inserting for the specific functional form of the utility of the two consumer types.

$$\frac{d}{dC}u(C) = \frac{d}{dC}\frac{1}{1-s}C^{1-s}$$

$$\Rightarrow u'(C) = C^{-s}$$

Now since the specific funtional form is indifferent of consumer type, we can insert this into (6) to get the specific solution.

$$\frac{C_2}{C_1} = R^{1/s} \tag{7}$$

Now, how is the initial wealth allocated?

I determines the investment in the long-term paper in period $t = 0$ and subsequently,

$1 - I$ determines the investment in the short term paper.

$$\frac{C_2}{C_1} = R^{1/s}$$
$$C_1 = \frac{1-I}{\lambda}$$
$$C_2 = \frac{RI}{1-\lambda}$$
$$(7) \Rightarrow \frac{\frac{RI}{1-\lambda}}{\frac{1-I}{\lambda}} = R$$
$$\frac{RI}{1-\lambda}\frac{\lambda}{1-I} = R^{\frac{1}{s}}$$
$$\frac{1-I}{RI} = \frac{\lambda}{1-\lambda}R^{-\frac{1}{s}}$$
$$\frac{1}{I} = \frac{\lambda}{1-\lambda}R^{-\frac{s-1}{s}} + 1$$
$$I = \frac{1}{\frac{\lambda}{1-\lambda}R^{-\frac{s-1}{s}} + 1} < 1 \tag{8}$$

Will there be any liquidation?

Only if late consumers have an incentive to withdraw their funds early, will there be liquidation. Otherwise, the law of large numbers ensures that the investment in the short run paper is sufficient to cover the demand from early consumers as (2) holds. The question thus becomes whether the contract is incentive compatible. In order for the solution to be incentive compatible, $C_2 \geq C_1$. We see that $\frac{C_2}{C_1} = R^{1-s}$. Since $R > 1, C_2 > C_1 \forall s$. Hence the solution is incentive compatible and there will be no liquidation.

Problem 2

An uneven distribution can be optimal, because in optimum, we have to consider the marginal allocation. On the margin, in optimum, we should allocate C_1 and C_2 in such a way that the marginal decrease in utility for early consumers by investing one additional unit, should at least (or in optimum - exactly) be compensated by the increase in utility from investing that one unit. This means that the payoff from investing one unit, R, must equal the increase in utility for late consumers needed to compensate the loss for the early consumers. Hence, uneven distributions may be optimal.

It is easy to see from (7) that:

$$\lim_{s\to\infty} \frac{C_2}{C_1} = \lim_{s\to\infty} R^{\frac{1}{s}} = 1$$

Hence, when $s \to \infty$ the optimal consumption profile is equality between late and early consumers.

Problem 3

Assume perfect competition, then the maximization problem is:

$$\max_{C_1,C_2} \lambda u(C_1) + (1-\lambda)u(C_2)$$
$$\text{s.t.}$$
$$(1-\lambda C_1)R = (1-\lambda)C_2 \tag{9}$$

Where the new constraint is the *zero-profit* constraint, and the new efficiency locus. Assume also that:

$$\frac{m(N)C_1}{N} = N(1-I) \Rightarrow N \to \infty$$
$$\lim_{N\to\infty} \frac{m(N)C_1}{N} = \lambda$$

It is now clear that the first-order conditions yield the same result as before. Hence C_1^*, C_2^* can be realized as an equilibrium with a banking sector having a zero-profit constraint under perfect competition. A final requirement is that late consumers believe that a sufficient fraction of late consumers will not withdraw their funds early. This point will be elaborated in the next problems.

Problem 4

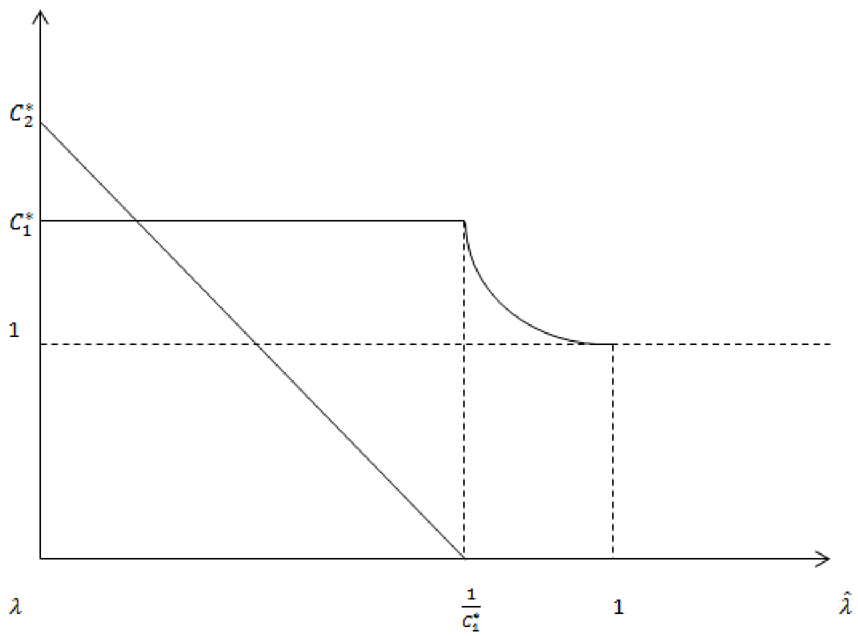

Figure 1: Nash equilibria with and without bank-run

From Figure 1 we see that there exists two Nash-equilibria in this model with these specifications. One Nash-equilibrium is the "good" one, where (C_2^*, C_1^*) is consumed.

This equilibrium will only exist until some late consumers start withdrawing their money early, that is, as long as $\hat{\lambda}$ is such that $C_2 \geq C_1$. If not, the late consumers will start withdrawing early, and less and less is paid out to the remaining consumers. Now we end up in a situation where everyone forces the bank to liquidate and, ultimately, the second Nash-equilibrium is reached where all consumers behave as early consumers and everyone gets 1 unit at time $t = 1$.

Problem 5

Case 1
Say $L = 1$, then:

$$\frac{1}{p} = R > 1 > C_2^*$$

$\Rightarrow$ Not a Nash-equilibrium.

Case 2

$$\exists L' : \frac{L'}{p} \leq C_2^*$$

$\Rightarrow$ A Nash-equilibrium still exists.

Now the late consumers can, in $t = 1$, liquidate their contract with the bank and invest their money in the bond market instead. If they liquidate, they receive 1 unit which can be used to buy $\frac{1}{p}$ bonds. 1 unit wealth invested in period 1, yields pR units wealth in period 2 for the consumer. Thus, the budget constraint is simply $1 = pR$, which can be rewritten as $\frac{1}{p} = R$.

If $R > C_2^*$, then there will be no banking solution as late consumers will earn more by liquidating their assets and investing them in the bond market. Due to the concavity of the utility function, we have that $\frac{d}{dc}(C^* u'(C)) < 0$, which we see holds in our case as $\frac{d}{dc}(C^* u'(-s)) = \frac{d}{dc}(C^(1-s)) = (1-s)C^{-s} < 0 \quad \forall s > 1$. Thus, $u'(1) \cdot 1 < u'(R) \cdot R \Rightarrow \frac{u'(1)}{u'(R)} > R$. We know that an efficient solution requires equality between the marginal rate of substitution (left-hand side) and the marginal rate of transformation (right-hand side). This can only be achieved if C_1 is increased above 1 so that $u'(C_1)$ decreases, which means that $C_1^* > 1$. From the efficiency locus, we then see that C_2^* must be reduced so that $R > C_2^*$. Thus if a bond market is introduced, and $L = 1$, there will be no good Nash-equilibrium from the banking solution. All late consumers will buy bonds, which means that the bank in period 1 will be emptied and C_2^*, C_1^* cannot be implemented.

However, the existence of a Nash-equilibrium depends crucially on the liquidation value. We see that for a value $L = L'$, $\frac{L'}{p} \leq C_2^*$, a Nash-equilibrium will exist.

Problem 6

As we see from Figure 2, the existence of multiple (2) Nash-equilibria depends on the amount of anxious individuals. Say the amount is such that the red line is the number of anxious individuals, then we clearly see that two Nash-equilibria still exists, as this

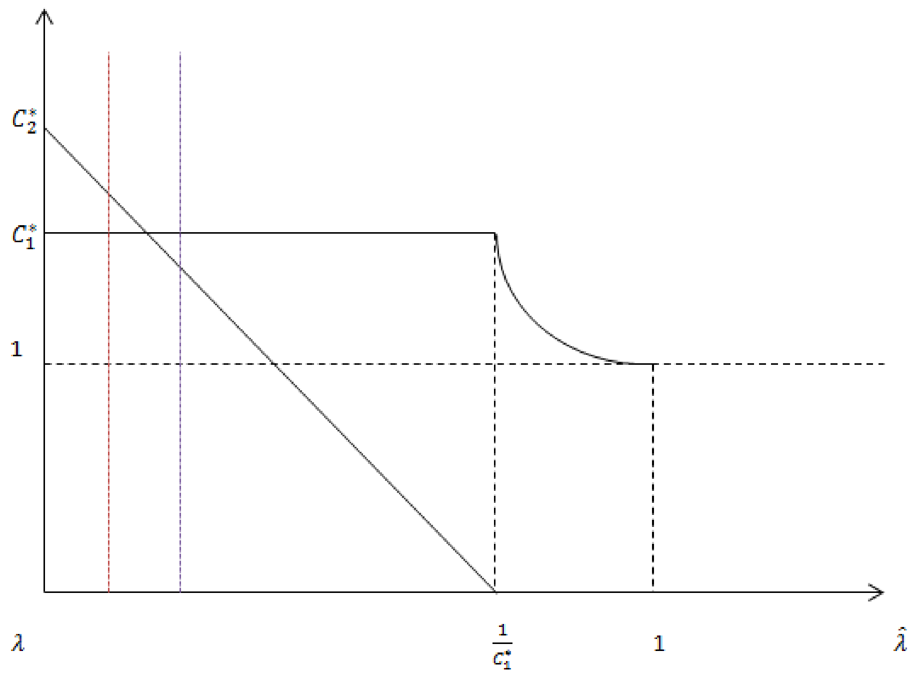

Figure 2: Nash equilibria with and without bank-run - Anxious individuals

is similar to "moving the vertical axis". If the purple line is the amount of anxious individuals, then clearly only the "bad" Nash-equilibrium exist.